Imperial College London

Lecture 13

Revision on Signals & Systems

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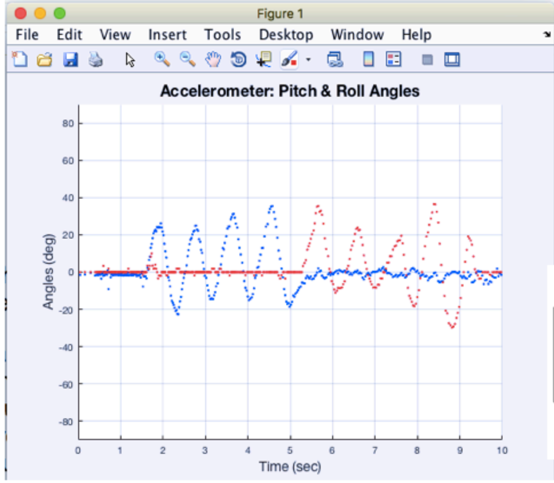
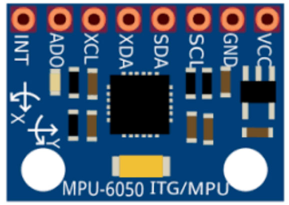
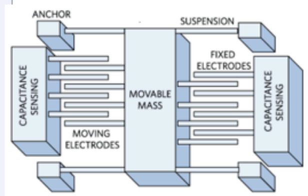
In this lecture, we will look back on all the materials we have covered to date. Instead of going through previous lecture materials, I will focus on what you have learned in the laboratory sessions, going from Lab 3 backwards to Lab 0. I will then relate the laboratory experiment to the theory covered during the lectures.

Lab 3: IMU basics - accelerometer



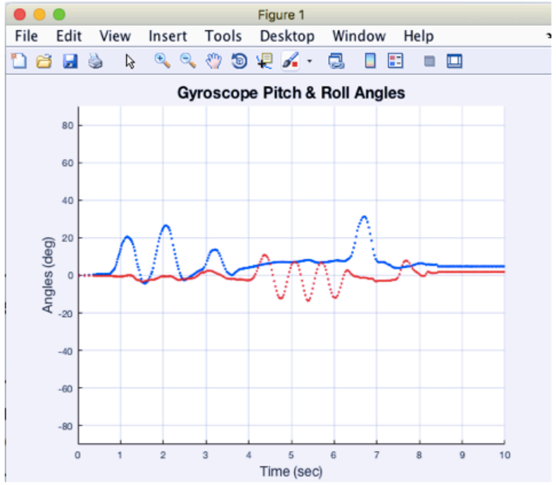


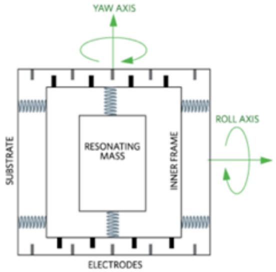


Question to ask yourself:

1. What is the basic principle of a MEM accelerometer?
2. What does such a device measure?
3. Why can an accelerometer derive the tilt angles (pitch and roll angles)?
4. Why, when using an accelerometer to measure tilt angles, you tend to have unwanted “noise”? What causes the noise?
5. The signal produced by the angular tilt and the noise have different characteristics. What are their differences?

Lab 3: IMU basics - gyroscope

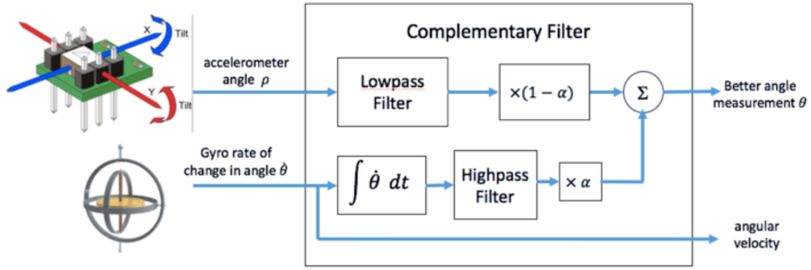




Question to ask yourself:

1. What does a gyroscope measures?
2. How can you derive tilt angle from gyroscope's measurements?
3. Why measuring angles from a gyroscope reading insensitive to linear motion? What is the implication of this property?
4. Why measuring angles from a gyroscope reading always produce drift?
5. What is the difference in the characteristic of angle measure by a gyroscope (ignoring the drift) and that of the drift?

Lab 3: Complementary Filter used with IMU



angle $\theta_{new} = \alpha \times (\theta_{old} + \dot{\theta}\, dt) + (1-\alpha) \times \rho$

where
- α = scaling factor chosen by users and is typically between 0.7 and 0.98
- ρ = accelerometer angle
- θ_{new} = new output angle
- θ_{old} = previous output angle
- $\dot{\theta}$ = gyroscope reading of the rate of change in angle
- dt = time interval between gyro readings

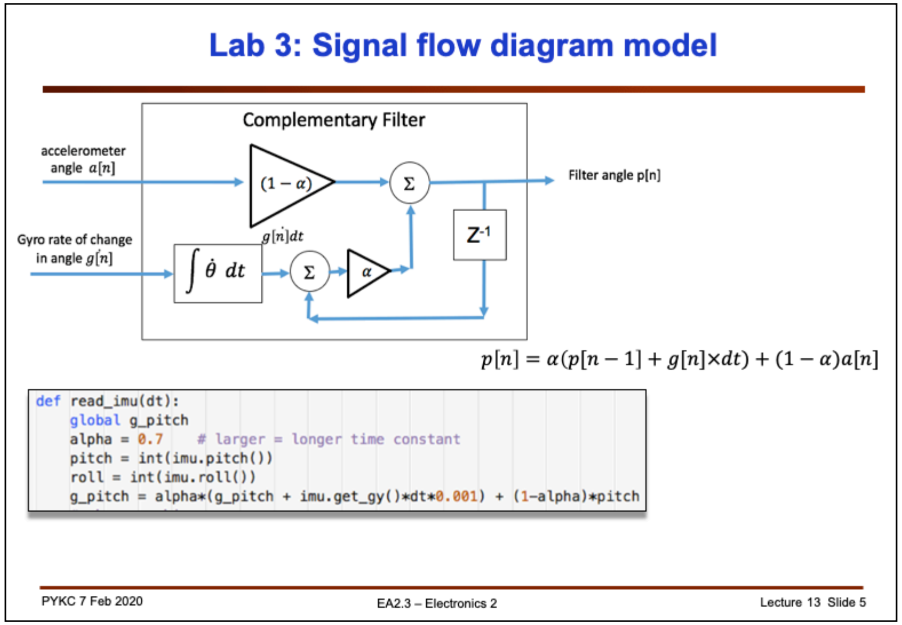
Remember that you implemented some kind of filter in Lab 3 to produce a much better angle measurement as shown here.

Now let us apply what we have learned to the Complementary filter used in the IMU measurements for the pitch and roll angles.

The block diagram is something that we have covered in Lab 3. The accelerometer angle is noisy, and we want to low pass filter (average) this to remove the noise due to movement.

The gyroscope measurement has drift (dc error, or slow changing value). We want to remove this via high pass filter.

We therefore derive the output pitch/roll angle, but by combining the readings from the accelerometer and the gyroscope.

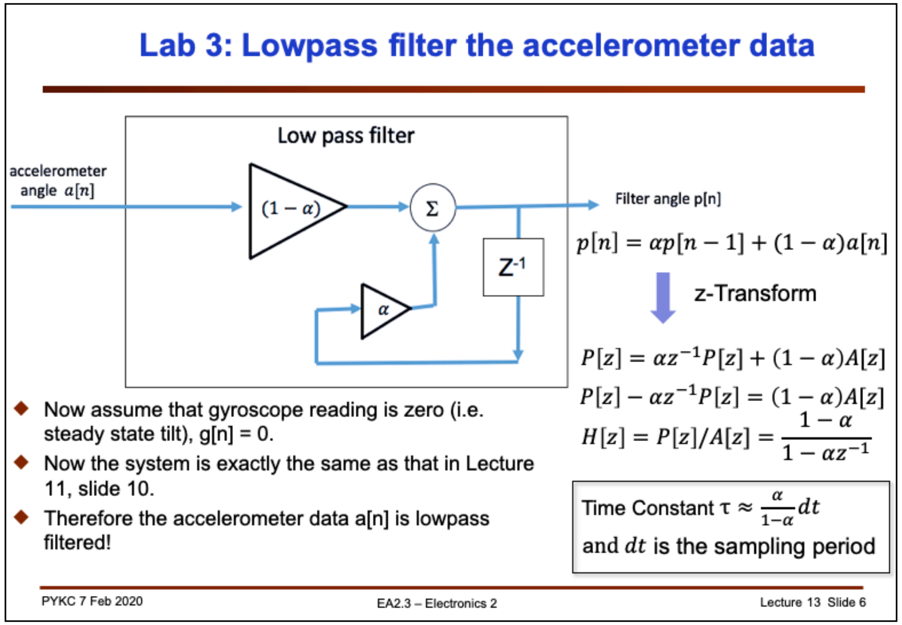


Before we consider how this filter works, let us cast the system in terms of difference equation. Shown here is the same set up as last slide, but now expressed as a difference equation.

The Micropython code to implement this is shown here. The equivalent version in Matlab for both pitch and roll angles is:

```
[p, r] = pb.get_accel();
[x, y, z] = pb.get_gyro();
dt = toc;
tic;
% integration for gyro angles
gx = max(min(gx+x*dt,pi/2),-pi/2);
gy = max(min(gy+y*dt,pi/2),-pi/2);

% complementary filtered angles
angle_x = alpha*(angle_x + x*dt) + beta*r;
angle_y = alpha*(angle_y + y*dt) + beta*p;
```



Let us for now assume that the gyroscope data is 0. The accelerometer data is applied to the discrete system as shown. Here we can immediate write down the difference equation as: $p[n] = \alpha p[n-1] + (1-\alpha)a[n]$

This is exactly the same structure as the IIR filter we have seen two slides earlier (α=0.8).

Now if we take the z-transform of the difference equation, remembering that:

$$Z\{p[n-1]\} = P[z]z^{-1}$$

We get:

$$P[z] = \alpha z^{-1}P[z] + (1-\alpha)A[z]$$

$$H[z] = P[z]/A[z] = \frac{1-\alpha}{1-\alpha z^{-1}}$$

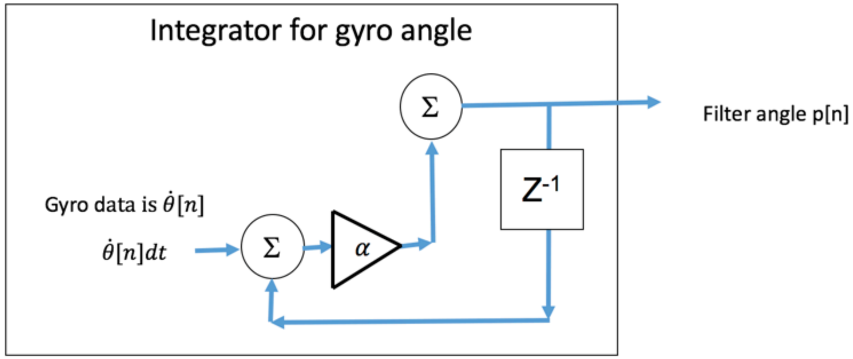
This is the discrete time transfer function of a typical low pass filter.

One important parameter of this filter is its **time constant** τ. It can be shown that if the sampling period (i.e. 1/fs) is dt, then

$$\text{Time Constant } \tau \approx \frac{\alpha}{1-\alpha} dt$$

Assuming that we have a sampling frequency of 100Hz, dt = 10ms and α = 0.8, the time constant is around 40msec. In other words, noise shorter than 40msec duration will be removed. If you want longer time constant, reduce value of α.

Lab 3: Integrating the gyroscope reading



$$p[n] = p[n-1] + \dot{\theta}[n]dt$$

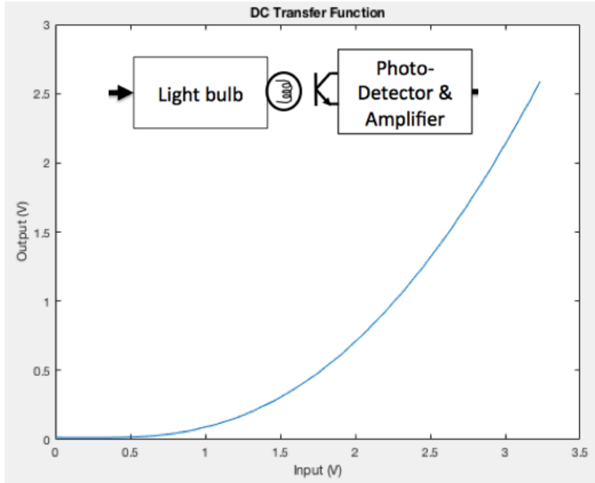
Now let us ignore the accelerometer signal, and only retain the gyroscope signal.

The integration process can be represented as a signal flow diagram and difference equation here. Assume that $\alpha = 1$, then p[n] is the integral of the gyro data.

Question to ask:

1. Why is this prone to drift if the gyroscope has an offset error?
2. The drift error is a constant offset k, if $\alpha < 1$, what will the effect of this error on p[n] over a long time?
3. Why is this equivalent to highpass filter?

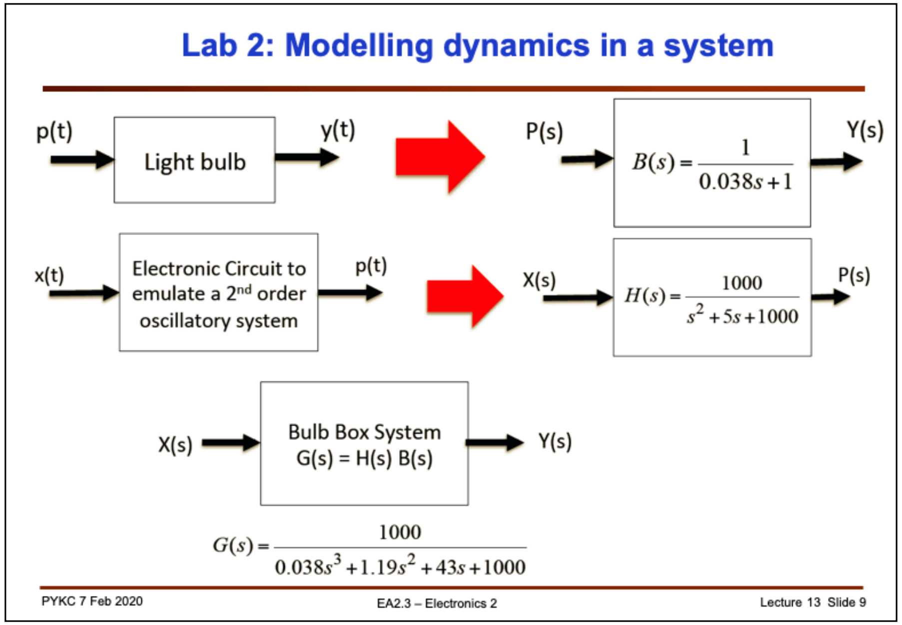
Lab 2: Light bulb DC characteristics



Lab 2 is all about characterisation of a system.

Questions to ask yourself:

1. What is meant by the DC characteristic of the system?
2. Why is the lightbulb system has such a characteristic? Is it linear? If not, what is it? Why is it this shape?



Questions to ask yourself:

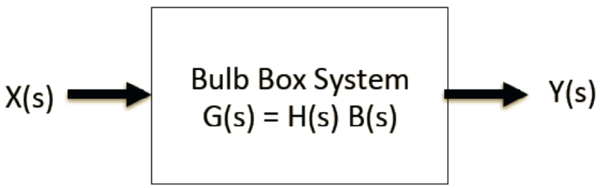
1. Considering just the light bulb without the extra electronic circuit. What is the order of this system? How would you expect such a system respond to a step input? Why?

2. What is the time constant of this light bulb circuit?

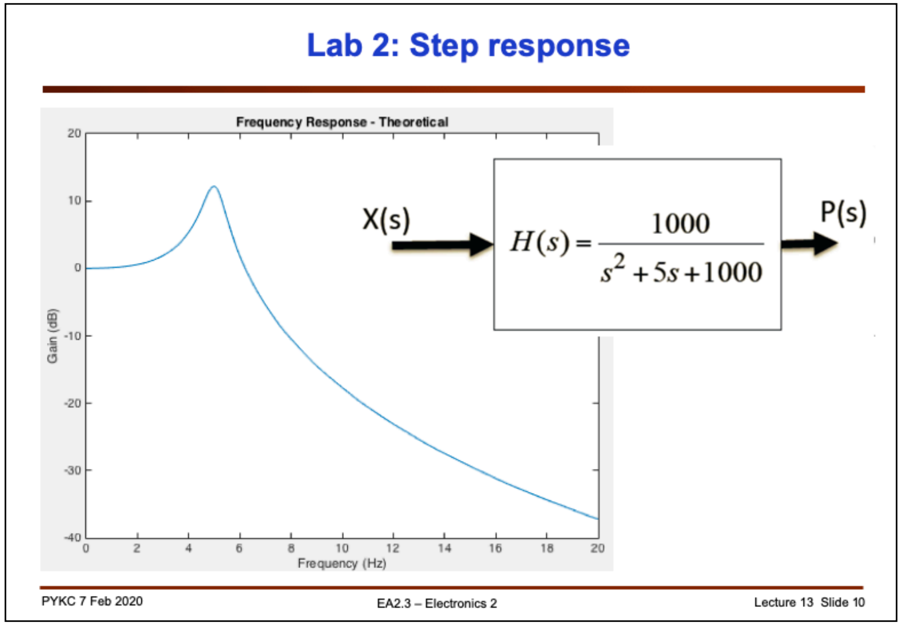
3. We add the extra electronic circuit before the light bulb. What order is this electronic circuit?

4. What is the DC gain of this model? Why?

5. The overall system transfer function is G(s). Why is this as shown here?



$$G(s) = \frac{1000}{0.038s^3 + 1.19s^2 + 43s + 1000}$$

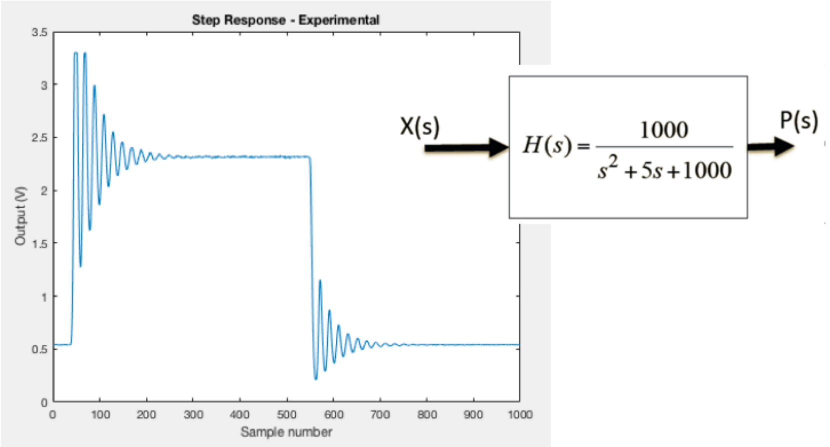


The dynamic behaviour of the light bulb box is dominated by the 2nd order electronic circuit with H(s) as shown.

Questions to ask yourself:

1. What is the resonant frequency of this system?
2. When measuring the frequency response, why did you use small signal amplitude?

Lab 2: Frequency Response of a second order system



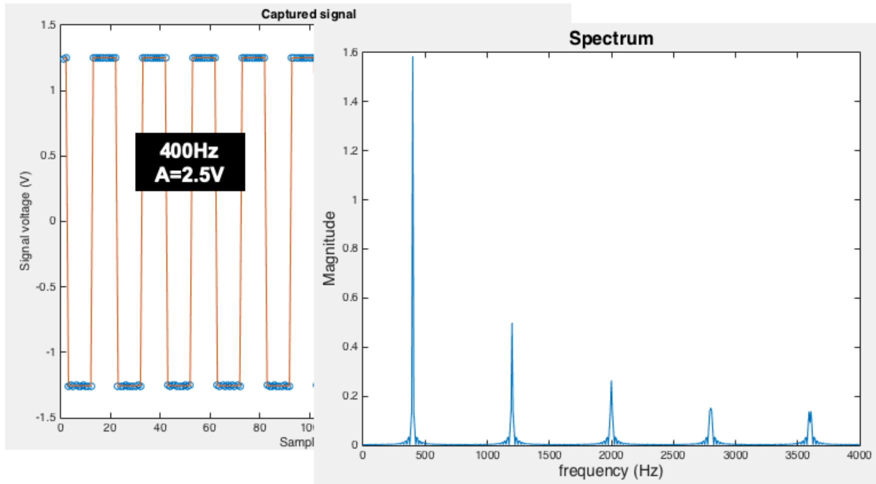
$$H(s) = \frac{1000}{s^2 + 5s + 1000}$$

This is the response of the system to square wave input.

Your questions:

1. What is meant by step response of a system?
2. What determines the frequency of the oscillation in this waveform?
3. Is this system highly, critically or lowly damped?
4. Which coefficient in the transfer function of the 2nd order system determines the resonant frequency?
5. Which coefficient in the transfer function determines the damping factor?
6. Why is the amplitude of oscillation larger for rising part than falling part of the square signal?

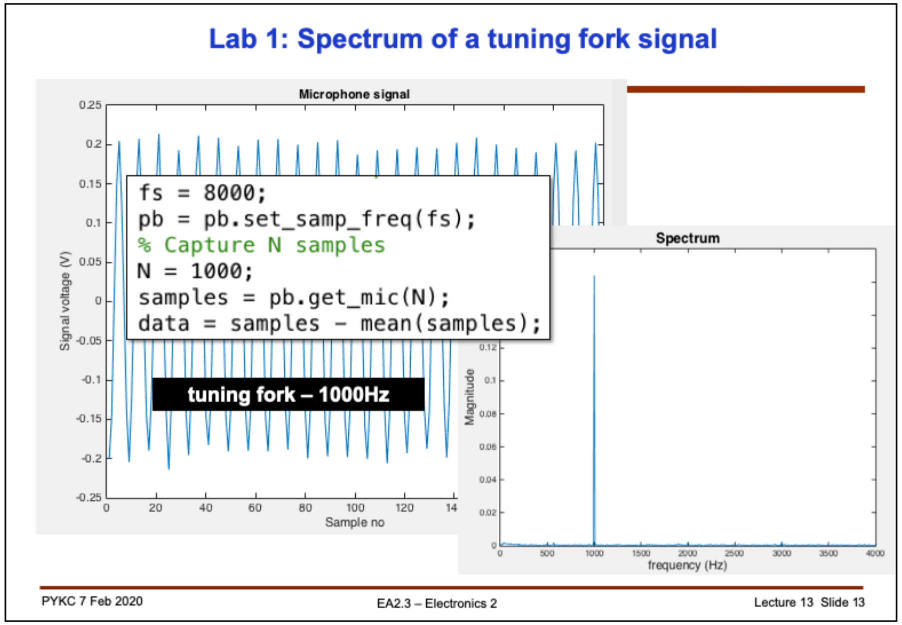
Lab 1: Spectrum of a square wave



Lab 1 is all about frequency and time domain representation of signals

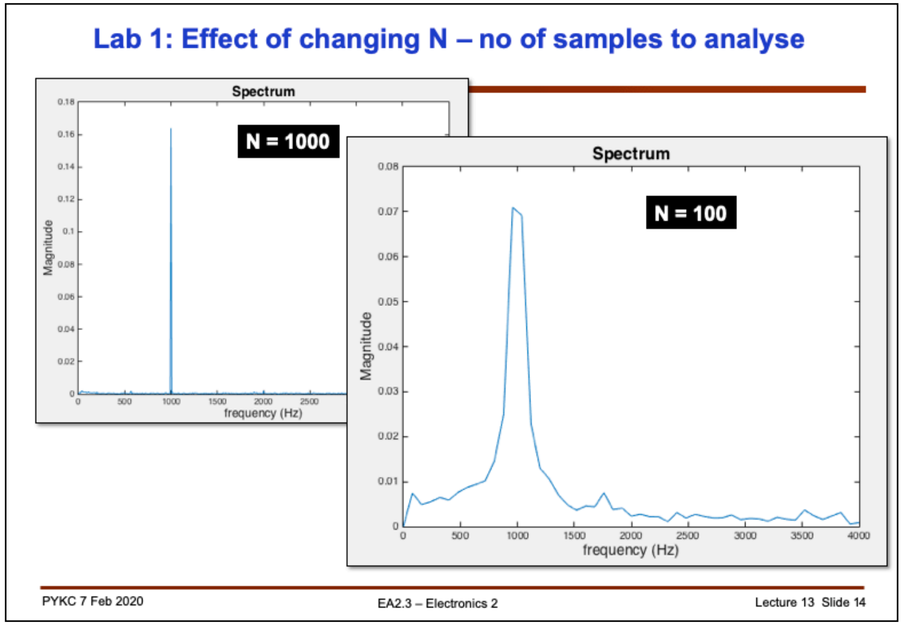
Your questions:

1. Why is the spectrum of a square wave looks like this?
2. How can one object the frequency spectrum of a signal?
3. What do you expect the spectrum of a sine wave and triangular wave to look like?



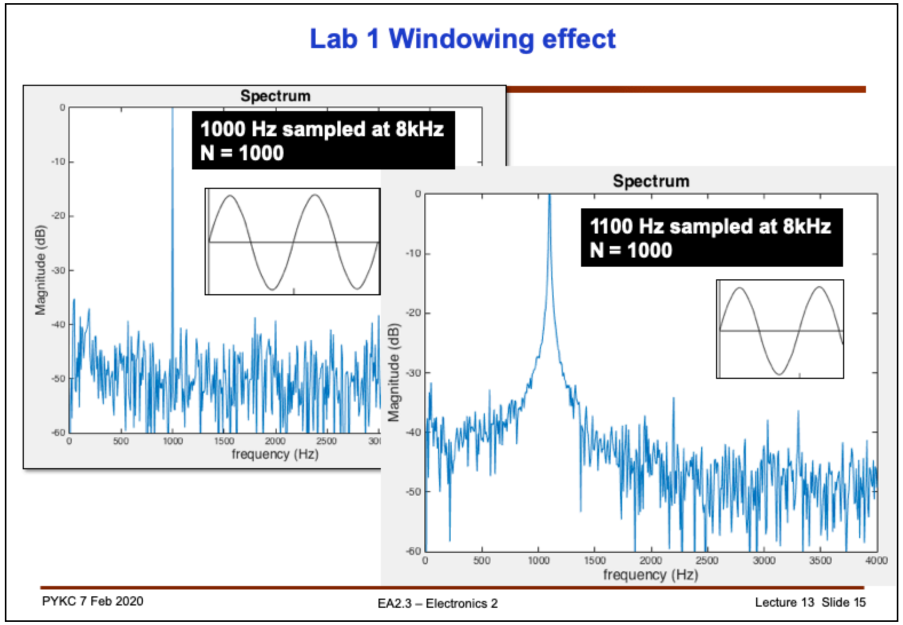
Questions to ask yourself:

1. What was the sampling frequency used for capturing this audio signal?
2. How frequency is the peak if we generate a tuning fork frequency at 4500 Hz? Why?
3. What do we call this phenomenon? Why is this bad?
4. What can one do to avoid this bad thing from happening?



Questions:

1. What happens if you change to number of signal samples for FFT from 1000 samples to 100?
2. How do you determine the frequency step in the spectrum?



You were sampling the audio signal at 8kHz and taking 1000 samples to perform FFT on.

Questions to ask yourself:

1. The spectrum for 1000Hz tone and for 1100 Hz tone are shown here. Why are they so different?
2. What is this effect called?
3. How can one reduce the impact of this effect?